

Characterization of the power and efficiency of Stirling engine subsystems



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HIGHLIGHTS

- We review experimental data from a V160 engine developed for cogeneration.
- We also investigate the V161 solar engine.
- The possible margin of improvement is evaluated for each subsystem.
- The procedure is based on similarity models and thermodynamic models.
- The procedure may be of general interest for other prototypes.

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ABSTRACT

The development of systems based on Stirling machines is limited by the lack of data about the performance of the various subsystems that are located between the input and output power sections. The measurement of some of the variables used to characterise these internal subsystems presents difficulties, particularly in the working gas circuit and the drive mechanism, which causes experimental reports to rarely be comprehensive enough for analysing the whole performance of the machine. In this article, we review experimental data from a V160 engine developed for cogeneration to evaluate the general validity; we also investigate one of the most successful prototypes used in dish-Stirling systems, the V161 engine, for which a seemingly small mechanical efficiency value has been recently predicted. The procedure described in this article allows the possible margin of improvement to be evaluated for each subsystem. The procedure is based on similarity models, which have been previously developed through experimental data from very different prototypes. Thermodynamic models for the gas circuit are also considered. Deduced characteristic curves show that both prototypes have an advanced degree of development as evidenced by relatively high efficiencies for each subsystem. The analyses are examples that demonstrate the qualities of dimensionless numbers in representing physical phenomena with maximum generality and physical meaning.

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1. Introduction

Features of the Stirling engine that are of greater interest for current applications are most likely the ability to adapt to various energy sources and to provide not only mechanical or electrical energy but also deliver thermal energy to its cold sink with low levels of noise and chemical pollution.

Systems based on Stirling engines contain various subsystems connected between the input and output power sections. Power is supplied by a heat exchanger adapted to the characteristics of

the energy source. The useful power is frequently obtained by an electrical generator driven by the engine mechanism and, in cogeneration applications, through a heat exchanger powered by the engine's cold source.

To characterise each subsystem, a number of variables must be measured, preferably by direct measurement under real operating conditions. However, the measurement of some variables presents difficulties, particularly in the working gas circuit and the drive mechanism because these subsystems are not directly connected to the input and output power sections. Consequently, it is rare that experimental reports are comprehensive enough, which hinders the validation of simulation procedures and the development of applications.

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Notation

[illegible]

The University of Oviedo and the IK4-TEKNIKER research centre have collaborated since 2007 on the development of Stirling engines capable of being used for solar energy conversion [1] and micro-combined heat and power (CHP) [2]. Since the beginning of this collaboration, it was considered interesting to analyse the performance of engines potentially useful for these applications using dynamic similarity models previously developed from experimental data on indicated power and mechanical losses observed in benchmark prototypes of very diverse characteristics: General Motors GPU-3 engine operating with helium and hydrogen, Philips M102C engine operating with air, United Stirling P40 engine operating with hydrogen, helium and nitrogen, Kockums V4275R Mark II engine operating with helium, Thermomotor 360-15 engine operating with hydrogen, Yamanokami 1 and Yamanokami 2 low-temperature differential (LTD) engines operating with air, Eco-boy SCM81 engine operating with helium and nitrogen, Mitsubishi and Daihatsu large-bored engine operating with helium, Vølund SM-1 engine operating with helium, Kolin's and Gros' LTD engines, Stirling's engine of 1818, etc. [3–5]. Such models have proven to be useful for design using scaling techniques [6,7], but have also provided analysis criteria, for example, to delimit the power produced by the original Stirling air engine of 1818 [8], to evaluate

predictions of simulation programs [9–11], to carry out comparative studies among empirical models [12,13], and to define operating ranges in control systems [14].

From a practical standpoint, experimental tests on Stirling engines are considered sufficient when at least the electrical and thermal powers are measured in the output section. Generally, such tests are performed for various mean pressure values of the working gas, which is often the main control variable. However, to evaluate the improvement margins of the subsystems it is desirable to obtain their individual efficiencies as a function of as many parameters and operating variables as possible. The rotational frequency of the engine is an essential performance variable, but it is generally only varied across a small range in grid-connected systems, which hinders analyses. In this article, the internal subsystems of a Stirling unit are characterised from measurements made at the input and output sections. The experimental data presented refer mainly to a V160 Stirling engine developed for cogeneration, but an important objective of the article is to draw conclusions applicable to one of the most successful prototypes used in dish-Stirling systems, the V161 engine, which had its mechanical efficiency questioned by a recent study [15]. To the best of our knowledge there is not any publication about other

systems for which the five major subsystems (combustion subsystem, gas circuit, drive mechanism, electrical subsystem and cooling subsystem) have been characterized experimentally. Using dimensionless variables facilitates the procedure, which may also be of general interest for other prototypes.

2. Experimental data

The V160 Stirling engine has evolved since its original development by United Stirling AB, Sweden, in the late 1960s. Development was later transferred to Stirling Power System, USA, which manufactured a number of versions before reaching commercial quality with the “G” series in 1989. The version known as V160-DMA is a Stirling cogeneration system that belongs to the “E” series [16]. Schlaich, Bergmann & Partner, Germany, bought a license from Stirling Power Systems and the manufacturing license was then transferred to SOLO Kleinmotoren GmbH, which produced commercial versions of the V160 engine in recent decades. The V161 Stirling engine is a V160 unit with modifications intended to reduce production costs, which is adaptable for solar applications based on the dish-Stirling concept [17]. Current models are marketed by Clean Energy, Sweden.

Experimental tests of the V160-DMA unit operating with natural gas as fuel and helium as the working fluid have been published [18]. In these tests, the engine subsystems (Fig. 1) were monitored to obtain both electrical and thermal output power, as well as the corresponding efficiencies.

The measured data were tabulated in series corresponding to 6 values of the water flow at the secondary cooling circuit, decreasing from $G_w = 1.95 \text{ m}^3/\text{h}$ (series “a”) to $G_w = 0.45 \text{ m}^3/\text{h}$ (series “f”). Each series contains data obtained for 11 values of the main control parameter, namely the mean working gas pressure, which was varied from $p_m = 3.7 \text{ MPa}$ to $p_m = 12.5 \text{ MPa}$. An additional engine control is obtained through the regulation of the cooling outflow temperature T_c , which is somewhat lower than the cooler wall temperature, while the maximum temperature of the working gas is maintained at a near-constant of $625 \pm 1^\circ\text{C}$. The operation of the asynchronous electrical machine causes the rotational frequency to stay near-constant as well, between the idle value of

1503 rpm at $p_m = 3.7 \text{ MPa}$ to 1518 rpm at $p_m = 12.5 \text{ MPa}$. Fig. 2 shows that, for each series, electrical and thermal power measurements are linearly proportional to p_m at both the input or output sections.

In addition to power and efficiency data, the tables also include measurements of the temperatures T_c , T_{cw} and T_{hw} at the primary and secondary cooling circuits for each series. Fig. 3 shows that these temperatures have a high degree of linearity as a function of p_m . From Figs. 2 and 3, it is deduced that varying the water flow G_w at the secondary cooling circuit does not provide a wide range of power control, because the trend lines lead to similar power values for each mean pressure value. This observation is in agreement with results obtained by Gheith et al. [19], who observed that flow rate has less significant effect in power brake performance than heating temperature and charge pressure.

While a detailed series of environmental air temperature measurements was not provided, the authors reported that they oscillated between 19 and 26°C .

3. Analysis of the external combustion subsystem

The performance analysis that will be described in the following sections requires experimental knowledge of the heat power absorbed by the engine heater. In steady operation, the combustion subsystem efficiency η_{comb} can be obtained by means of the balance of power transferred through the control volume (Fig. 1) that includes the subsystems formed by the Stirling engine, the primary cooling circuit and the alternator, that is to say:

$$\eta_{comb} = \eta_e + \eta_t + k_L \quad (1)$$

where the k_L factor has been introduced to consider heat losses through the control volume's casing.

This k_L factor is not available from the experimental data, but it can be estimated. Table 1 shows the mean combustion efficiency values, their standard deviations, and the mean k_L factors, which have been obtained for each series, assuming that losses to environmental air can be represented by a heat transfer coefficient of $10 \text{ W}/(\text{m}^2^\circ\text{C})$, that the control volume casing has a surface area

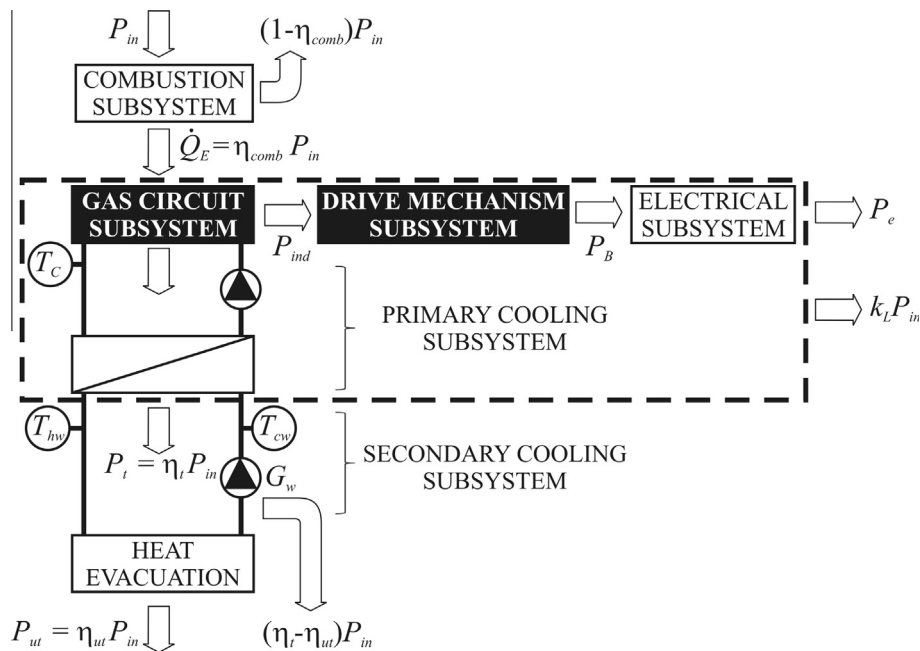


Fig. 1. Power flow between the subsystems.

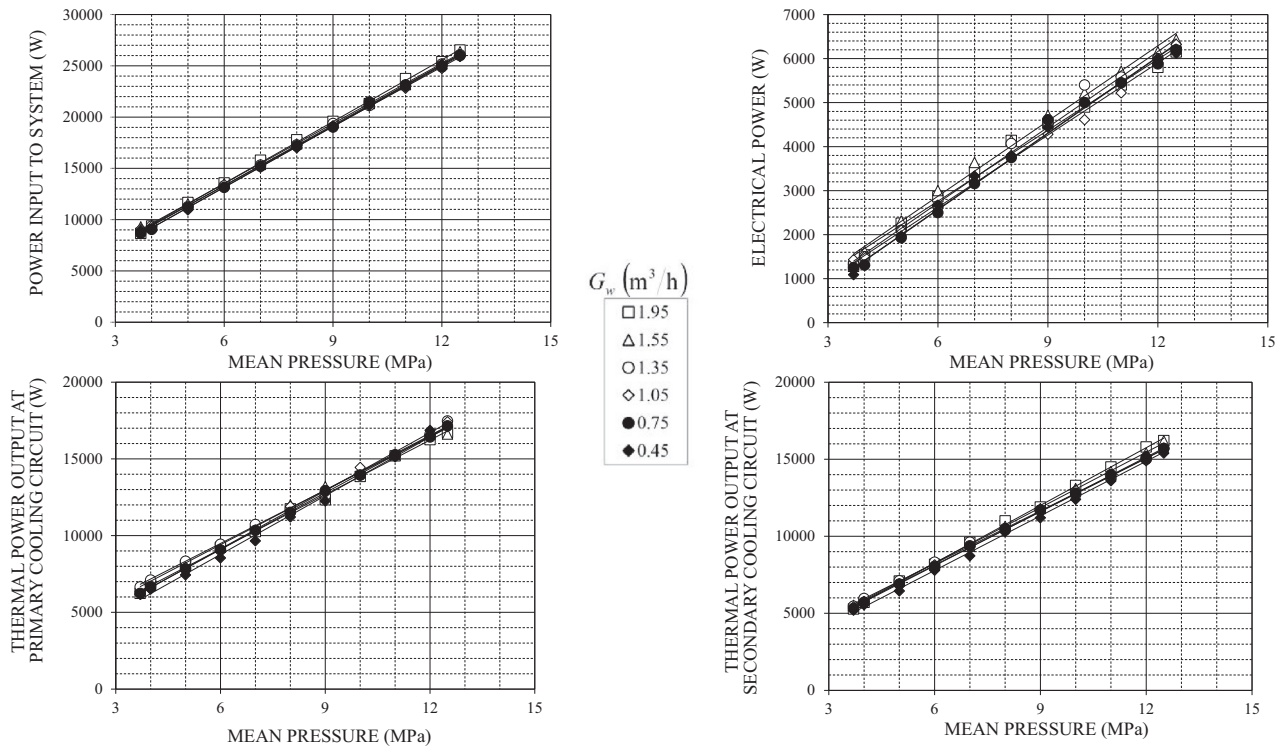


Fig. 2. Variation of input and output powers as a function of the mean pressure.

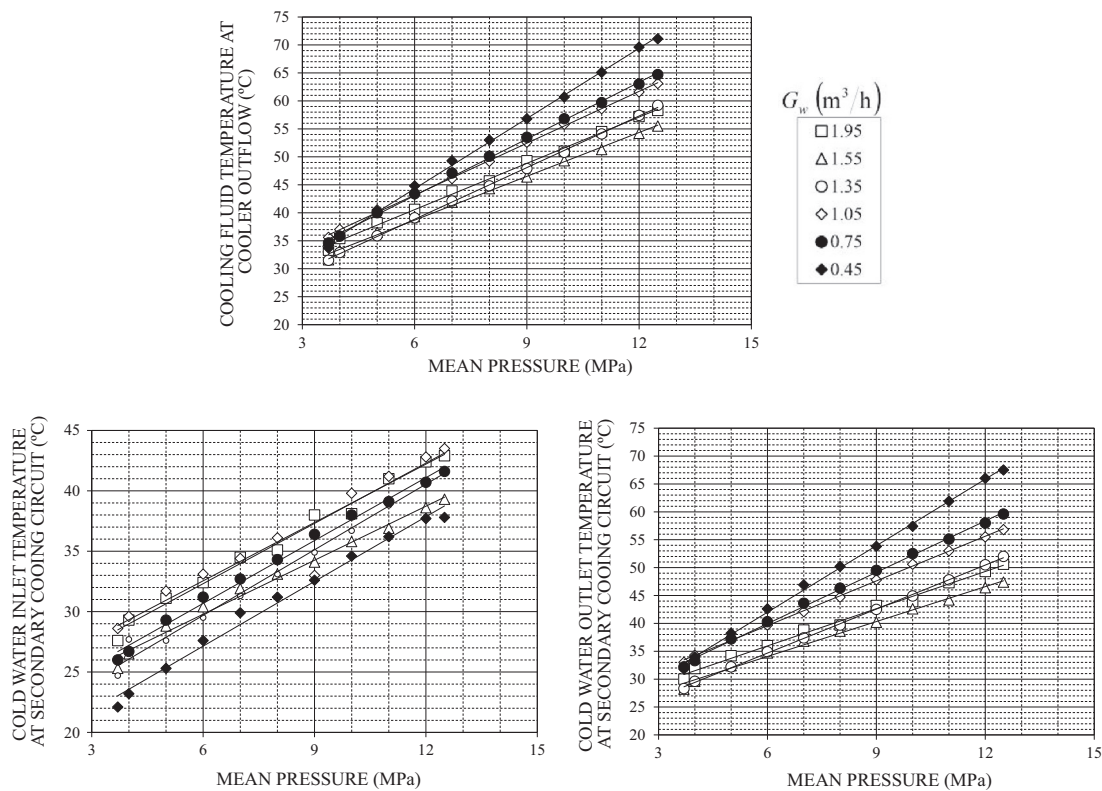


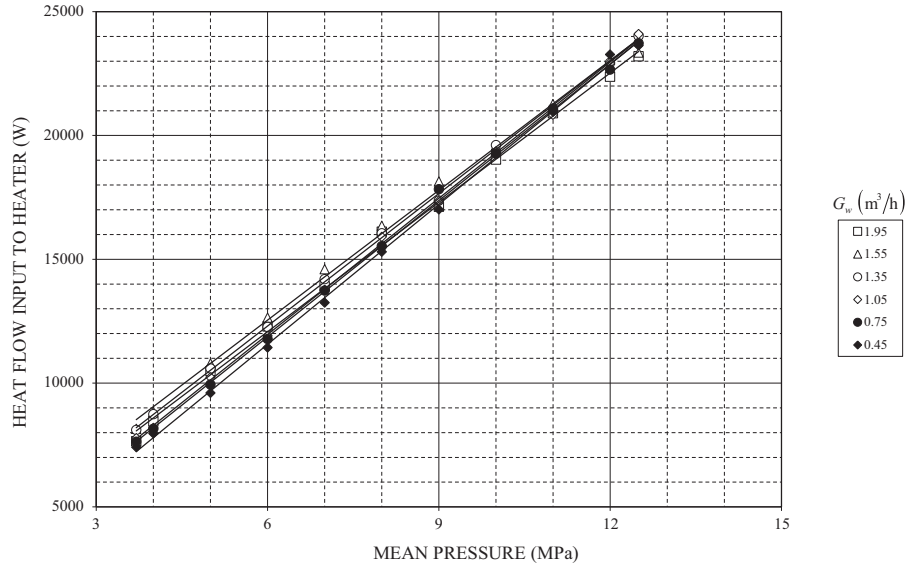
Fig. 3. Variation of temperatures in the primary and secondary cooling circuits as a function of the mean pressure.

of 0.75 m^2 [20] at a temperature 5°C higher than the cooling fluid temperature T_C at the cooler outflow, and that the environmental air temperature is 20°C . These assumptions lead to mean η_{comb} values of approximately 90%, with relatively low standard deviations

for each series. It is deduced that η_{comb} is almost independent of the mean pressure and therefore the input and output powers; thus, the heat power $\dot{Q}_E$ absorbed by the engine heater is also linearly proportional to p_m for each series (Fig. 4).

Table 1Mean values of the combustion subsystem efficiency and k_L factor.

	Series “a”	Series “b”	Series “c”	Series “d”	Series “e”	Series “f”
Mean efficiency	0.89 ± 0.01	0.92 ± 0.02	0.92 ± 0.01	0.90 ± 0.02	0.90 ± 0.01	0.89 ± 0.03
Mean k_L factor	0.014 ± 0.001	0.013 ± 0.001	0.013 ± 0.001	0.015 ± 0.001	0.015 ± 0.001	0.017 ± 0.001

**Fig. 4.** Variation of heat power absorbed by the engine heater as a function of the mean pressure.

4. Analysis of the internal subsystems

For each series, it can be written that:

$$\eta_e = \eta_{alt} \eta_{mec} \eta_{ind} \eta_{comb} \quad (2)$$

Thus, once k_L has been estimated, the experimental data will allow the electrical efficiency to be deduced based on the heat power absorbed by the engine heater, i.e., the product $\eta_{ind} \eta_{mec} \eta_{alt}$ for each value of the mean cycle pressure p_m . These data can be used as a starting point for the analysis of the subsystems' performance, with the goal of obtaining the subsystem efficiencies as a function of the main engine operating variables, i.e., mean pressure, rotational engine frequency and temperatures.

Series “e” has been selected as the case study. This series' data are listed in Table 2, with an additional four columns for the original measurements reported by Lista [18]. The values of n_s are estimates deduced from the aforementioned operational range of the rotational frequency, while the last three columns are a consequence of Eq. (1).

Table 2Series “e” experimental data ($G_w = 0.75 \text{ m}^3/\text{h}$, $T_E = 625 \text{ }^\circ\text{C}$).

P_m (MPa)	n_s (rpm)	T_c ($^\circ\text{C}$)	T_{hw} ($^\circ\text{C}$)	T_{cw} ($^\circ\text{C}$)	P_{ut} (W)	P_e (W)	P_m (W)	η_t (-)	P_t (W)	$\dot{Q}_e$ (W)	$k_L P_m$ (W)	η_{comb} (-)
3.7	1505.0	34.6	32.2	26.0	5310	1250	8660	0.72	6235	7629	144	0.88
4.0	1505.4	35.8	33.3	26.7	5720	1310	9040	0.74	6690	8153	153	0.90
5.0	1506.9	40.0	37.2	29.3	6900	1930	11,180	0.70	7826	9941	185	0.89
6.0	1508.4	43.4	40.3	31.2	8050	2500	13,140	0.69	9067	11,777	210	0.90
7.0	1509.9	47.1	43.6	32.7	9380	3160	15,200	0.68	10,336	13,734	238	0.90
8.0	1511.4	50.1	46.3	34.3	10,500	3750	17,190	0.67	11,517	15,528	260	0.90
9.0	1512.8	53.5	49.5	36.4	11,700	4610	19,020	0.68	12,934	17,829	286	0.94
10.0	1514.3	56.8	52.5	38.0	12,800	5000	21,470	0.65	13,956	19,266	311	0.90
11.0	1515.8	59.7	55.1	39.1	14,000	5450	23,140	0.66	15,272	21,055	332	0.91
12.0	1517.3	63.0	58.0	40.7	15,000	5880	24,880	0.66	16,421	22,658	357	0.91
12.5	1518.0	64.7	59.6	41.6	15,700	6200	25,990	0.66	17,153	23,723	370	0.91

Fig. 5 shows evidence that the V160 system reaches η_e^* values which are slightly higher than 26% at a mean pressure ranging from 10 to 12.5 MPa.

4.1. Semi-empirical models for the analysis of subsystem performance

Organ [21] and Prieto et al. [3,22] independently introduced the following functional relationship to express the dimensionless indicated power of kinematic Stirling engines:

$$\zeta_{ind} = f(\tau, K, \lambda_1, \dots, \lambda_k, \mu_{dec}, \mu_{de}, \mu_{dr}, \mu_{dc}, \mu_{dce}, \alpha_{ce}, \alpha_{cr}, \alpha_{cc}, \lambda_{he}, \lambda_{hr}, \lambda_{hc}, \gamma, N_s, N_{TCR}, N_p, N_{MA}) \quad (3)$$

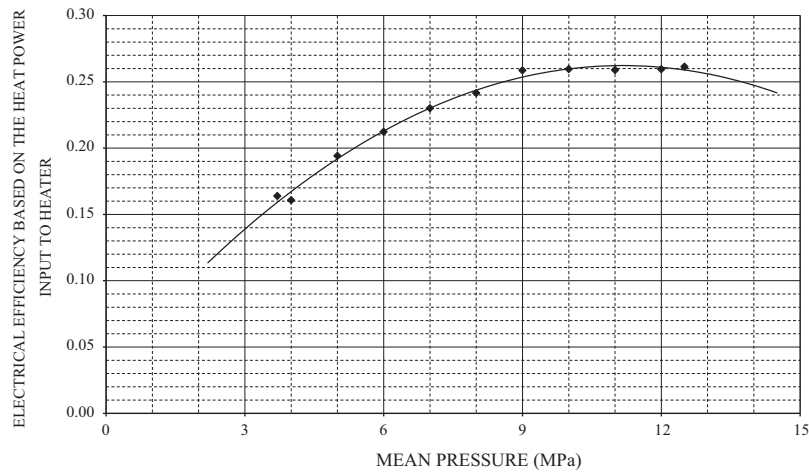
Prieto et al. [4] proposed the following approach that explicitly states the influence of the engine speed on the dimensionless indicated power:

$$\zeta = \zeta_0 - \Phi N_{MA} - \Psi N_{MA}^2 \quad (4)$$

In this equation, ζ_0 denotes the quasi-static dimensionless indicated work per cycle that represents the theoretical prediction, assuming

Table 3Data for the analysis of heat power consumption (V160 engine, T_{we} 625 °C).

P_m (MPa)	n_s (rpm)	T_c (°C)	N_{SG} (–)	N_{MA} (–)	N_P (–)	ζ_0 (–)	$\zeta_0/(1-\tau)$ (–)	$\dot{Q}_E/(p_m V_{sw} n_s)$ (–)
3.7	1505.0	34.6	7.75E+09	0.00182	1.41E+07	0.2058	0.313	0.4225
4.0	1505.4	35.8	8.37E+09	0.00182	1.52E+07	0.2052	0.313	0.4175
5.0	1506.9	40.0	1.05E+10	0.00181	1.89E+07	0.2029	0.311	0.4069
6.0	1508.4	43.4	1.25E+10	0.00180	2.26E+07	0.2011	0.311	0.4013
7.0	1509.9	47.1	1.46E+10	0.00179	2.62E+07	0.1991	0.309	0.4007
8.0	1511.4	50.1	1.67E+10	0.00179	2.98E+07	0.1975	0.308	0.3961
9.0	1512.8	53.5	1.87E+10	0.00178	3.33E+07	0.1957	0.308	0.4039
10.0	1514.3	56.8	2.08E+10	0.00177	3.68E+07	0.1940	0.307	0.3924
11.0	1515.8	59.7	2.29E+10	0.00176	4.04E+07	0.1924	0.306	0.3894
12.0	1517.3	63.0	2.49E+10	0.00176	4.38E+07	0.1907	0.305	0.3838
12.5	1518.0	64.7	2.59E+10	0.00175	4.55E+07	0.1899	0.304	0.3856

**Fig. 5.** Variation of electrical efficiency based on the heat power absorbed by the engine heater, as a function of the mean pressure.

an ideal cycle without any losses caused by leakage, thermal or mechanical irreversibilities. This thermodynamic concept can be calculated for every engine if data are available about the heat sources' temperatures, the dead volumes and the drive mechanism, according to the following functional relationship:

$$\zeta_0 = f(\tau, \kappa, \lambda_1, \dots, \lambda_k, \mu_{dec}, \mu_{de}, \mu_{dr}, \mu_{dc}, \mu_{dcc}) \quad (5)$$

The coefficients Φ and Ψ of indicated power losses characterise irreversibilities inherent to the operation at practical engine speed, as for example the pumping losses [4]. Those factors do not depend on the engine speed and can be accurately calculated when the maximum indicated power $P_{ind,max}$ and the corresponding velocity $n_{s,max}$ are known by means of the following expressions:

$$\Phi = \frac{2\zeta_0 - 3\zeta_{max}}{N_{MA,max}} \quad (6)$$

$$\Psi = \frac{2\zeta_{max} - \zeta_0}{N_{MA,max}^2} \quad (7)$$

Notice that $N_{MA,max}$ depends on the engine operating parameters and is inversely proportional to the indicated power loss coefficients, therefore it is an index of the development level of the gas circuit.

To calculate the ζ_0 values corresponding to the analysed operating points, geometrical data of the V160-DMA engine are needed. The geometrical parameters used in the present analysis are those available in the PROSA 2.4 software [23], which are consistent with the prototype version with a modified phase angle between pistons whose tests were reviewed by Lista [16]. This version has pistons with independent crank drive mechanisms, which allows

the phase angle to be adjusted from 90° to 105° and consequently the net swept volume V_{sw} to be varied from the 226 cc of early versions to approximately 195 cc.

The mechanical power losses P_{mec} can be calculated based on the following model [5]:

$$\zeta_{mec} = \alpha e^{-\beta/N_{SG}} + \chi + \frac{\delta}{N_{SG}} \quad (8)$$

In Eqs. (3), (4), and (8), both N_{MA} and N_{SG} may be interpreted as dimensionless forms of the engine's rotational frequency.

4.2. Analysis of the heat power consumption

At very low engine speeds, the indicated efficiency should tend to its well-known quasi-static value, namely:

$$\lim_{n_s \rightarrow 0} \eta_{ind} = 1 - \tau \quad (9)$$

Imposing this condition on η_{ind} and taking into consideration Eq. (4) leads to the following restriction for the dimensionless heat power supplied to the engine heater:

$$\lim_{n_s \rightarrow 0} \frac{\dot{Q}_E}{p_m V_{sw} n_s} = \frac{\zeta_0}{1 - \tau} \quad (10)$$

This constraint, not previously observed, suggests that the dimensionless heat power consumption $\dot{Q}_E/(p_m V_{sw} n_s)$ can be analysed based on dimensionless groups that are characteristic of the Stirling engine's performance. Table 3 shows the numerical values involved in the analysis, which has been performed under the assumptions that $T_{we} \approx 625$ °C and $T_{wc} \approx T_c$. Among several approaches considered to match the conditions of Eq. (10), the best adjustment to

Table 4Indicated power analysis for the V160 engine operating with helium at $T_{we} \approx 625$ °C.

P_m (MPa)	n_s (rpm)	Eq. (3) (W)	MARWEISS (W)	PROSA (W)	SNAP (W)	García-Granados (W)	Error (%)
3.7	1505.0	3017	2172	1592	3010	3085	2.2
4.0	1505.4	3253					
5.0	1506.9	4019					
6.0	1508.4	4780					
7.0	1509.9	5521					
8.0	1511.4	6259	5690	4208	6187	6410	2.4
9.0	1512.8	6976					
10.0	1514.3	7683	6998	5247	7579	7747	0.8
11.0	1515.8	8380					
12.0	1517.3	9060	8403	6134	8896	8880	−2.0
12.5	1518.0	9398					

the experimental data has been obtained by means of the following correlation, with RMSE = 0.0038 and CV(RMSE) = 4.15%:

$$\frac{\dot{Q}_E}{p_m V_{sw} n_s} = \frac{\zeta_0}{1 - \tau} + 8.871 N_{MA}^{0.101} N_p^{-0.230} \quad (11)$$

Fig. 6 shows a comparison between lines of constant N_{MA} based on Eq. (11) and experimental values of $\dot{Q}_E/(p_m V_{sw} n_s) - \zeta_0/(1 - \tau)$, which represents the increase in heat power consumption attributed to irreversibilities. It is observed that the fluctuations of this value range from 7.5% to 11%, mainly due to pressure variation. The linear shape at practical operating conditions is caused by the small variation in N_{MA} .

4.3. Analysis of indicated power

Because no data were available to apply to Eqs. (6) and (7), Φ and Ψ must be estimated from simulation software and experimental data.

The results of the commercial programs MarWeiss [24], PROSA [23] and SNAP [25], developed decades ago, were used, along with results provided by the author [26] of a simulation program recently developed under the Envirodish Project for the SOLO V161 engine operating with hydrogen [15].

Table 4 and Fig. 7 show comparisons between the different simulations for mean pressures of 3.7, 8, 10 and 12 MPa, assuming $T_{we} \approx 625$ °C for all cases. The values of indicated power calculated by means of Eq. (4) correspond to $\Phi = 20$ and $\Psi = 675$, which are the values that agree the most with the simulations provided by García-Granados, with relatively low percentage differences, which

are listed in Table 5. In these calculations, Φ and Ψ are assumed to not change meaningfully with variations of the temperature ratio τ and the dimensionless pressure number N_p , in accordance with previous experimental observations on very different prototypes [4,5]. Fig. 7 also shows that the calculations based on Eq. (4) are in agreement with the experimental measurements of indicated power conducted on a recently restored V160F Stirling engine, installed since the 1990s at CEDER-Soria, Spain [14]. SNAP values were observed to be close to the predictions of García-Granados' program, while those of MarWeiss and PROSA show greater differences.

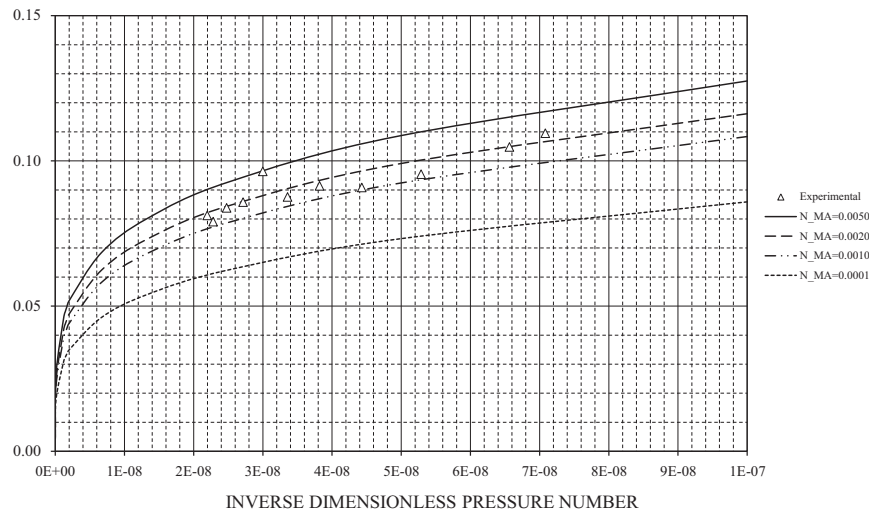
4.4. Analysis of mechanical power losses and alternator efficiency

Because the product of the mechanical efficiency and the alternator efficiency is determined after estimating the indicated power, obtaining each of these efficiencies can be attempted using Eq. (8) and data about η_{alt} .

Tests of the General Motors GPU3 engine suggested expressing the coefficients α , β , χ , δ of Eq. (8) by means of the following potential functions [5]:

$$\alpha, \chi, \delta = a_i \frac{N_m^{a_i}}{N_p^{a_i}} \quad \beta = \frac{a_l}{(\mu_l/\mu)^{a_m}}$$

where μ and μ_l are the viscosities of the working gas and lubricating fluid, respectively, at the reference temperature T_{wc} , $N_m = m_1 RT_{wc}/(p_m V_{sw})$ is the dimensionless number characteristic of moving masses in the drive mechanism, and m_1 is one of those masses, used as a reference.

**Fig. 6.** Increase in heat power consumption due to irreversibilities.

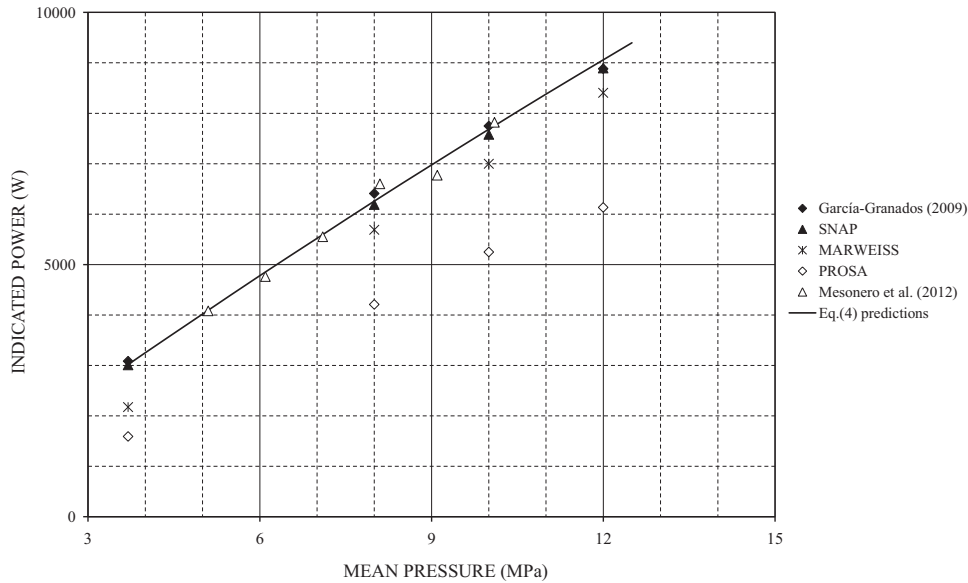


Fig. 7. Comparison of indicated power values for the V160 engine operating with helium at $T_{wE} \approx 625$ °C.

Table 5
Mechanical efficiency analysis for the V160 engine operating with helium at $T_{wE} \approx 625$ °C.

p_m (MPa)	n_s (rpm)	T_C (°C)	N_p (-)	N_{SG} (-)	$1/N_{SG}$ (-)	ζ_{mec} (-)	P_{mec} (W)	P_B (W)	η_{mec} (-)	η_{alt} (-)
3.7	1505.0	34.6	1.41E+07	7.75E+09	1.29E-10	0.066	1191	1826	0.605	0.684
4.0	1505.4	35.8	1.52E+07	8.37E+09	1.19E-10	0.062	1218	2035	0.626	0.644
5.0	1506.9	40.0	1.89E+07	1.05E+10	9.56E-11	0.054	1308	2711	0.675	0.712
6.0	1508.4	43.4	2.26E+07	1.25E+10	7.98E-11	0.048	1399	3381	0.707	0.739
7.0	1509.9	47.1	2.62E+07	1.46E+10	6.85E-11	0.044	1495	4026	0.729	0.785
8.0	1511.4	50.1	2.98E+07	1.67E+10	6.00E-11	0.041	1597	4662	0.745	0.804
9.0	1512.8	53.5	3.33E+07	1.87E+10	5.33E-11	0.039	1706	5270	0.755	0.875
10.0	1514.3	56.8	3.68E+07	2.08E+10	4.81E-11	0.037	1823	5860	0.763	0.853
11.0	1515.8	59.7	4.04E+07	2.29E+10	4.37E-11	0.036	1948	6432	0.768	0.847
12.0	1517.3	63.0	4.38E+07	2.49E+10	4.01E-11	0.035	2080	6979	0.770	0.842
12.5	1518.0	64.7	4.55E+07	2.59E+10	3.85E-11	0.035	2149	7248	0.771	0.855

Although the engine frequency is almost a constant in these series of measurements, the mean pressure variation is sufficient to express the experimental data as a function of the Stirling number $N_{SG} = p_m/(\mu n_s)$. This approach leads to a function for ζ_{mec} determined by a set of 11 coefficients, $a_1, \dots, a_{11}$. Each series contains 11 sets of measurements, but errors hinder the calculation of these coefficients as the solution of a system of equations. Therefore, it was necessary to estimate the coefficients of Eq. (8) by means of numerical calculus, taking into account the following information:

- An alternator efficiency of 92% at an electrical power of 9 kW has been reported for the V160F unit installed at CEDER-Soria, Spain, as well as efficiencies in the range of 75–80% at “low values” of electrical power [27].
- The recent restoration of this unit has allowed η_{alt} measurements to be obtained, which resolves the inaccuracy of the above information at low power levels [14].
- An efficiency of 92.5% at an electrical power of 10,850 W has been reported for a similar alternator coupled to the SOLO-V161 unit erected under the EnviroDish Project at the CNRS-PROMES laboratory in Odeillo, France [20].

Table 5 and Fig. 8 summarise the results of the best correlation obtained for the set of η_{alt} data, which can be expressed by means of the following equation, assuming $\mu_L = 0.3$ Pa s:

$$\zeta_{mec} = 0.500N_p^{-0.139}e^{-7.250 \cdot 10^{10}(\mu/\mu_L)^{0.020}/N_{SG}} + 0.580N_p^{-0.199} + 2.99 \cdot 10^8 N_p^{0.008}/N_{SG} \quad (12)$$

It must be noted that N_m has not been considered in the list of variables influencing this correlation, which implies that some numerical coefficients are dependent not only on engine parameters but also on N_m . However, it is thought that this simplified procedure is acceptable for the objectives of this article related to the analysis of mechanical power losses.

From Fig. 8 it is deduced that the correlation predicts an alternator efficiency of approximately 90% at an electrical power of 7550 W, which is comparable the value of 85% estimated for a similar alternator on the EuroDish SOLO V161 unit installed at the University of Seville, Spain, under the EnviroDish Project [15].

5. Discussion

Fig. 9 shows the different efficiencies of the V160 unit as a function of the characteristic dimensionless pressure number. When calculating the characteristic efficiency and power curves of Stirling engines, it is desirable to specify not only the operating variables but also the parameters that remain constant, because hypersurfaces would be needed for a complete representation of models such as Eq. (3). As for the figures analysed in the previous

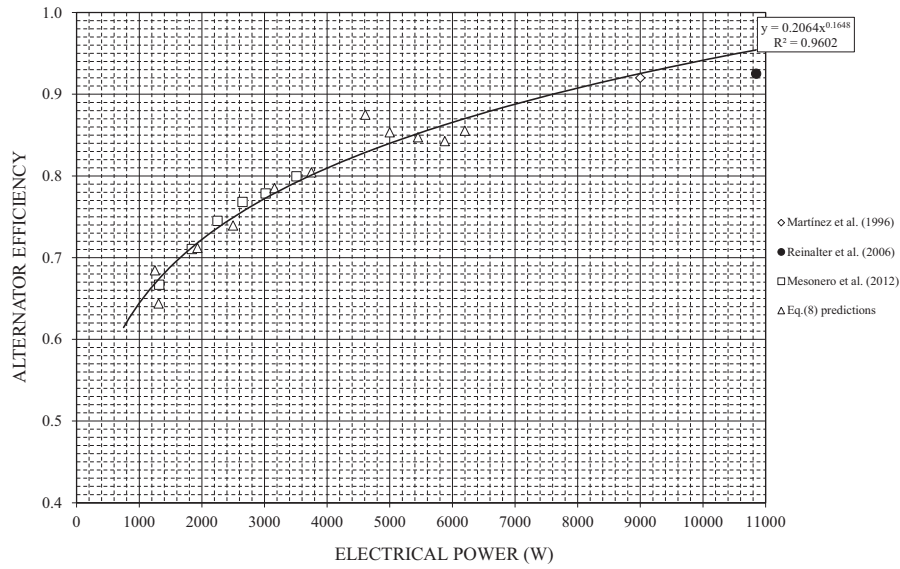


Fig. 8. Variation of the alternator efficiency as a function of the electrical power.

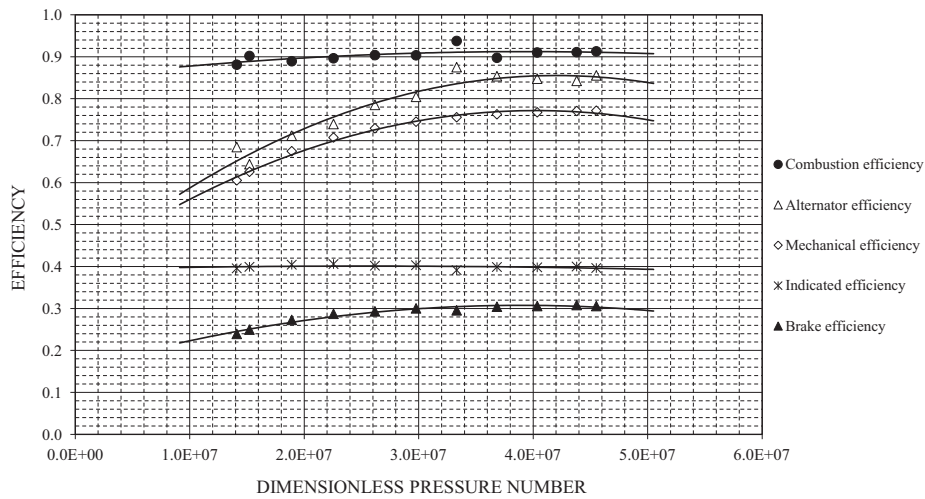


Fig. 9. V160 efficiencies as a function of p_N , operating with helium at $0.342 \leq \tau \leq 0.376$ and $N_{MA} \approx 0.0018$.

sections, in Fig. 9 the rotational frequency is approximately constant, but the temperature ratio varies almost 10% between the lowest- and highest-pressure points of each line. Thus those figures consist of projections of operating points located in different hyperplanes.

In addition to previous comments concerning the combustion subsystem efficiency and the alternator efficiency, it is observed that the indicated efficiency is approximately equal to 40% regardless of the value of N_p , which is influenced by the two control variables (p_m and T_C). It is also evident that the brake efficiency has small variations under medium and high pressures, where it reaches values slightly higher than 30%. The V160 engine reaches relatively high η_{mec} values compared to other prototypes, of approximately 77% during nominal operation in the experimental series examined, which has not been observed until now.

Therefore, the mean pressure variations do not result in large variations in indicated efficiency, as is the case with the dimensionless indicated power. In contrast, the mean pressure is influential in the mechanical efficiency. In agreement with Senft [28], we can say that the mechanical efficiency is affected by the operating

conditions of the gas circuit, showing that forced work is greater at low pressures for the V160 engine. Then the brake efficiency variations with the mean pressure are caused mainly by the mechanical performance characteristics.

Fig. 10 shows that efficiencies have higher variation with respect to the rotational frequency when the temperature ratio and average pressure remain constant. Despite there is one only experimental value of each efficiency for the considered values of τ and N_p , the curves can be obtained from the models' equations, provided that $\eta_{ind} = \zeta_{ind} / (\dot{Q}_E / p_m V_{sw} n_s)$ and $\eta_{mec} = (\zeta_{ind} - \zeta_{mec}) / \zeta_{ind}$. No experimental data are available to corroborate the curves at very low speeds, but the figure indicates, for example, that the engine could attain a brake efficiency increase of approximately 13% if it operates at 1000 rpm, which may be of interest from for possible control systems with speed regulation.

Figs. 11 and 12 show the dimensionless power curves under the same conditions assumed for the above efficiency characteristic curves. Because dimensionless power is proportional to torque, this type of curves is interesting not only for power analysis but also from the viewpoint of operational stability. Experimental

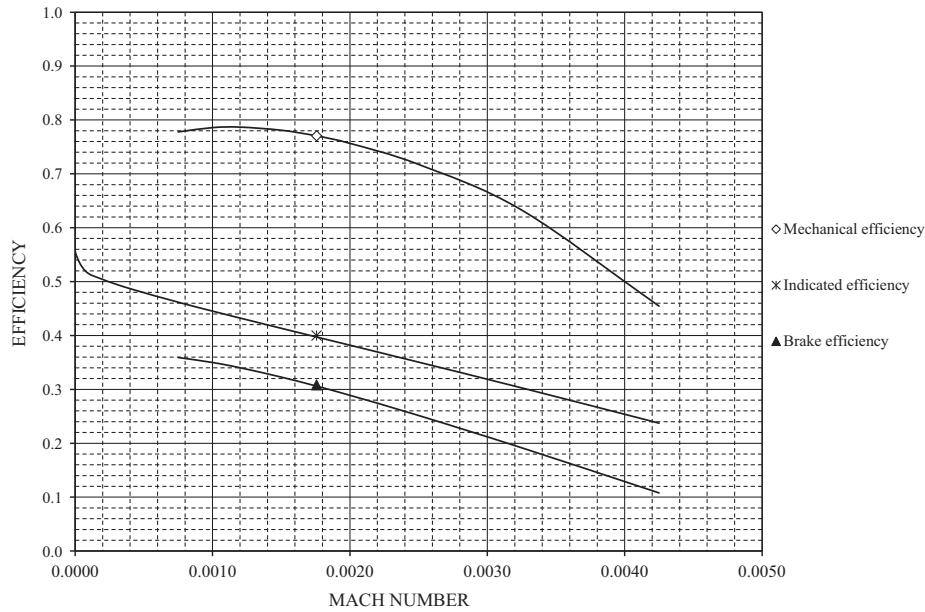


Fig. 10. V160 efficiencies as a function of N_{MA} , operating with helium at $\tau \approx 0.374.0$ and $N_p = 4.38 \cdot 10^7$.

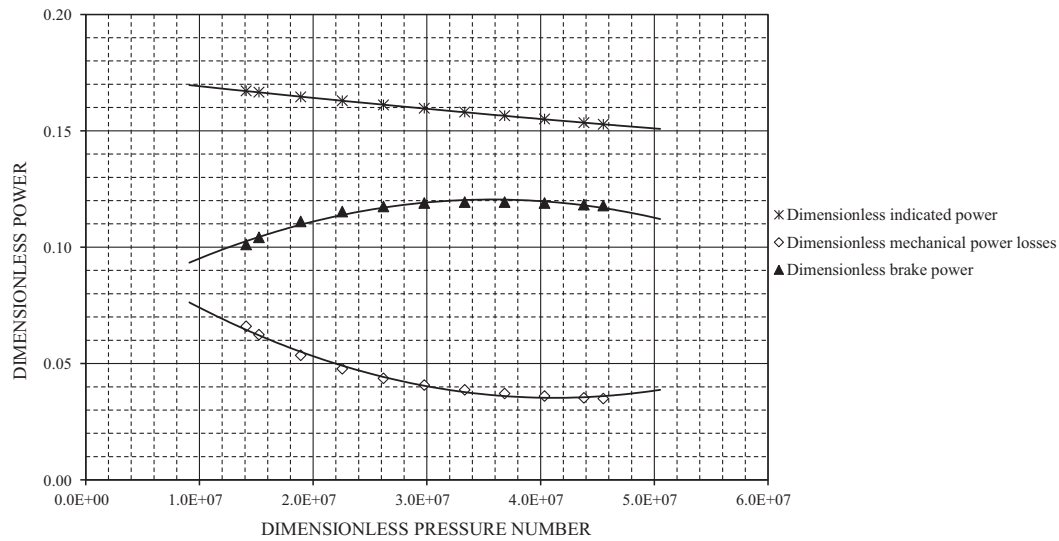


Fig. 11. V160 dimensionless powers as a function of N_p , operating with helium at $0.342 \leq \tau \leq 0.376$ and $N_{MA} \approx 0.0018$.

torque measurements of other recently developed engines [29,30] show trends that are consistent with the models applied in this article. The model of Eq. (8) is capable of explaining brake efficiency and torque decrease providing the mean pressure increases and the engine speed remains constant. Moreover, an increase in dimensionless indicated power with engine speed for temperatures and mean pressure unchanged may be observed in experimental engines likely due to leakage losses that occur at certain operating conditions.

In summary, the models of dimensionless power and efficiency can be adjusted to the V160 engine experimental data with an acceptable degree of accuracy. These models allow us to analyse the influence of each operating parameter that has been selected as a variable, especially if the rest of the parameters remain constant. This procedure allows the possible margin of improvement for each subsystem to be evaluated. The analytical equations of such models are often not linear, but they have the advantages of

being based on physical concepts and have fewer numerical coefficients than matrix models recently developed under the sponsorship of the International Energy Agency [31].

Another advantage of using models based on dimensionless numbers is that they allow the results to be generalised to other engines using dynamic similarity criteria.

In this way, the values of η_e^* shown in Fig. 5 are clearly in agreement with the value of 26.5% estimated for the V161 unit installed at the University of Seville, operating with hydrogen at a mean pressure of 11.22 MPa [15]. This result might be considered a consequence of the similarity between both prototypes; however, the V161 engine has geometric differences, mainly at the heater, due to its modification for a system supplied by solar energy. Moreover, the use of different gases also causes the dimensionless numbers characteristic of the operating point to be quite different. Those differences reduce the dynamic similarity and are influential on both the indicated power and efficiency.

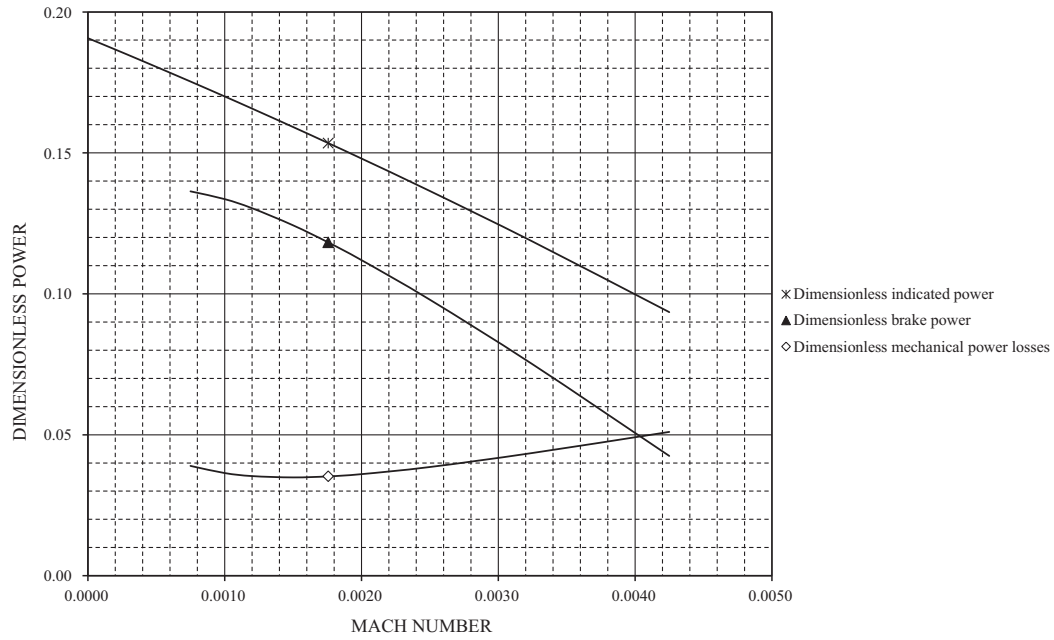


Fig. 12. V160 dimensionless powers as a function of N_{MA} , operating with helium at $\tau \approx 374.0$ and $N_p \approx 4.38 \cdot 10^7$.

Table 6
Geometric characteristics of the Stirling engines.

	V_{sw} (cm ³)	$\frac{V_{dR}}{V_{sw}}$ (-)	$\frac{V_{dL}}{V_{sw}}$ (-)	$\frac{V_{dC}}{V_{sw}}$ (-)	$\Sigma \mu_{dx}$ (-)	$\frac{r_{hR}}{L_R}$ (-)	$\frac{r_{hL}}{L_L}$ (-)	η_V (-)	$\frac{r_{hC}}{L_C}$ (-)	$\frac{A_{wR}}{V_{sw}^{2/3}}$ (-)	$\frac{A_{wL}}{V_{sw}^{2/3}}$ (-)	$\frac{A_{wC}}{V_{sw}^{2/3}}$ (-)
V160	194.55	0.209	0.328	0.190	1.729	0.00313	0.00064	0.61	0.00313	16.169	972.766	35.322
V161	194.55	0.326	0.353	0.190	2.005	0.00141	0.00093	0.69	0.00313	42.038	735.239	35.322

Table 6 compares the geometric characteristics of the heat exchangers of both engines. As stated before, the V160 engine data are contained in the PROSA 2.4 software, while the V161 data come from measurements made on the units installed at the University of Seville [26] and Odeillo [32]. The 24 tubes with lengths of 240 mm in the V160 engine heater were replaced in the solar receiver of the V161 engine by 78 tubes with lengths of 320 mm and a smaller diameter, with the advantages of creating a heater with more than double the heat exchange surface and favouring a uniform distribution of temperature, but with the disadvantages of increasing the dead volume and decreasing the hydraulic radius-to-length ratio.

An indicated power of 13,670 W and indicated efficiency of 48% were calculated for the V161 unit of the EnviroDish Project, operating with hydrogen at $p_m = 11.22$ MPa and $T_{wE} \approx 732$ °C [15]. It must be noted that the authors question the validity of this mechanical efficiency, which should be equal to 64.9% to attain the measured electric power of 7550 W with the estimated alternator efficiency of 85%.

As discussed below, this mechanical performance actually seems low because the V160 and V161 engines have similar drive mechanisms and Eq. (12) predicts that the V161 engine mechanical power losses would be of the same order as the ones for the V160 at high pressures (Fig. 13). Furthermore, the inconsistency is even greater when the higher alternator efficiency for the electric power measured predicted by Fig. 8, of approximately 90%, is considered.

It is also interesting to note that the lines in Fig. 13 have the appearance of Stribeck's curves, where friction increase at high pressures and low velocities may be caused by changes in the lubrication regime. It is also evident the analogy between the Hershey number and the inverse Stirling number, which has already

been argued as a foundation of the model of Eq. (8) [5,9]. Although there are no experimental evidences for values greater than 12.5 MPa, the trend lines of Figs. 5 and 9 indicate the possibility that the mechanical efficiency will decrease above a certain mean pressure value, consequently affecting the characteristic curves of brake and electrical efficiency. Similarly, the model of Eq. (8) may explain the decrease in brake power at relatively high mean pressures observed in other recent experiments already cited [30].

Eq. (4) provides a way to analyse whether the V161 engine can reach the indicated power of 13,670 W predicted by the simulation program, and then assess whether the mechanical efficiency should be higher.

First, with the geometrical data in Table 6 and the parameters corresponding to the operating point considered, listed in Table 7, the value of ζ_0 that is obtained for the V161 engine is greater than those obtained for the V160 at high pressure levels.

Furthermore, it is expected that the loss coefficients Φ and Ψ are lower for the V161 engine operating with hydrogen than for the V160 engine with helium because the following relationship was observed for the GPU-3 engine operating with both gases [7]:

$$\frac{\Phi, \Psi(H_2)}{\Phi, \Psi(He)} \approx \frac{46}{50} = 0.92$$

In addition, the differences between the Φ and Ψ coefficients of the V160 and V161 engines can be justified by geometrical reasons. As shown in Table 6, the ratio r_{hR}/L_R and the regenerator porosity are 45.3% and 13.1% higher, respectively, in the V161 engine than the assumed values for the V160. Because the indicated power losses usually are an order of magnitude higher in the regenerator than in the other heat exchangers [33], it is expected that the indicated power loss coefficients of the V161 engine are lower than

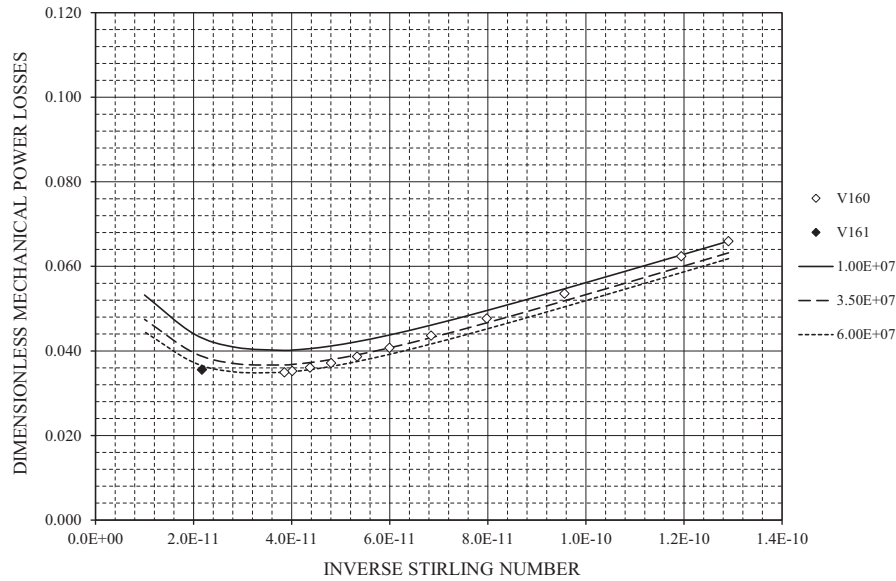


Fig. 13. Variation of dimensionless mechanical power losses as a function of the inverse N_{SC} with N_p as a parameter.

Table 7

Indicated power analysis for the V161 engine operating with hydrogen at $T_{WE} \approx 732$ °C.

P_m (MPa)	n_s (rpm)	N_{MA}	N_{SC}	N_p	T_c (°C)	ζ_0
11.22	1523	0.00128	4.61E+10	5.87E+07	55.3	0.2019

those of the V160 engine, even operating with helium. However, even with zero loss coefficients, 13,670 W cannot be reached using Eq. (4).

Whatever the origin of the discrepancies, it can be seen that Eq. (10) leads to $P_{mec} \approx 1970$ W for the above operating point of the V161 engine, as well as $P_B \approx 8389$ W, assuming $\eta_{alt} \approx 90\%$. Thus, $P_{ind} \approx 10,359$ W and $\eta_{mec} \approx 81\%$.

Eq. (4) is in agreement with this indicated power calculation using $\Phi \approx 13$ and $\Psi \approx 440$, which leads to $N_{MA,max} \approx 0.0060$. This value of $N_{MA,max}$ exceeds the results observed for other engines until now, which indicates that the V161 gas circuit is at an advanced developmental level, even if the indicated power is well below the predictions of the simulation program. Higher loss coefficients would result in a smaller indicated power, but in this case the mechanical efficiency or the alternator efficiency would have to be even greater.

The possibility of achieving $N_{MA,max} \approx 0.0060$ can be evaluated by experiments carried out with the EuroDish unit in Odeillo because the Φ and Ψ coefficients vary little with changes in temperature and mean pressure. In this case, $P_e \approx 10,850$ W was measured at a moderate direct normal irradiance level of 906 W/m² and low ambient temperature of -5 °C [20]. The engine operating point is not reported by the authors, but the values of $T_{WE} \approx 780$ °C, $T_{WC} \approx 40$ °C and $p_m \approx 14$ MPa are deduced from another publication related to the same experiments [34]. Substituting the values $N_{SC} \approx 5.72 \cdot 10^{10}$ and $N_p \approx 7.29 \cdot 10^7$ into Eq. (12) allows $P_{mec} \approx 2517$ W to be obtained for these operating conditions. Therefore, the assumption of $\eta_{alt} \approx 92.5\%$ leads to $P_{ind} \approx 14,247$ W, and consequently $\eta_{mec} \approx 82\%$.

A quasi-static dimensionless indicated power of $\zeta_0 \approx 0.2172$ can also be computed for the operating conditions of the Odeillo unit, so that Eq. (4) allows $P_{ind} \approx 13,884$ W to be obtained at $n_s \approx 1530$ rpm for the coefficients of indicated power loss estimated

at the Seville unit, i.e., $\Phi \approx 13$ and $\Psi \approx 440$. The difference of 2.6% between the indicated power calculations may be due to small errors in the measurements of operating conditions or, more most likely, in the estimations of η_{alt} , Φ or Ψ . However, it can be concluded that the estimates should be close to reality and the Odeillo data are consistent with a value of $N_{MA,max}$ slightly greater than 0.0060.

Therefore, the indicated power of the V161 engine may exceed the value of 13,670 W predicted by García-Granados' simulation program, but not at the operating conditions considered for the unit installed in Seville. It seems unlikely that the discrepancy is due to a fundamental flaw of the simulation program, as in previous sections it has been seen that its predictions proved acceptable for the V160 engine, but the simulations may have been performed with inadequate assumptions about the geometric characteristics. For example, the authors mention [15] that they adopted geometrical data specified by Organ [35] for the V160 engine. Those data refer to an engine version with a heater dead volume somewhat lower than the value specified in Table 6 for the V160 engine, and, more importantly, with a drive mechanism that has a phase angle of 90° , and therefore $V_{sw} \approx 225$ cm³ and different instantaneous volumes.

In short, the analyses show that both prototypes have an advanced degree of development as evidenced by relatively high efficiencies for each subsystem.

6. Conclusions

V160. engine tests have been performed with small variations in rotational frequency, but with mean pressure variations sufficient to achieve the research objective of deducing power and performance characteristics for the internal subsystems.

The combustion subsystem shows very uniform efficiencies for all series of measurements, with values of approximately 90%.

The internal subsystems have been analysed by means of similarity models which have been previously developed through experimental data from very different prototypes and are consistent with recent measurements in other experimental engines. Thermodynamic models for the gas circuit are also taken into account. Model uncertainties have been evaluated by considering experimental data for indicated power and alternator efficiency.

For the V160 engine operating with helium at a mean pressure range from 10 to 12.5 MPa, values of electrical efficiency, based on the heat power absorbed by the engine heater, are slightly higher than 26%, which is in agreement with calculations for the V161 unit installed at the University of Seville, operating with hydrogen at a mean pressure of 11.22 MPa.

The increase in heat power consumption which is attributed to irreversibilities, varies from 7.5% to 11%, mainly due to the variation in pressure. The curves of heat power absorbed by the engine heater as a function of the mean pressure show a linear shape at practical operating points due to the small variation of the characteristic Mach number.

Operating with helium at a mean pressure of 12 MPa and a temperature ratio of 0.374, the V160 engine can achieve an indicated efficiency of 40%, with a mechanical efficiency of 77%. Therefore, it produces 5875 W of electrical power at approximately 1517 rpm, with an alternator efficiency of approximately 84%.

The electric power of 7550 W measured at 1523 rpm on the V161 engine installed in Seville, operating with hydrogen at a mean pressure of 11.22 MPa and temperature ratio of 0.327, is compatible with a mechanical efficiency of 81% and an alternator efficiency of 90%. These values correspond to an indicated power of approximately 10,360 W, significantly lower than that estimated in previous publications, most likely due to errors in the geometric parameters assumed.

The electric power of 10,850 W measured at 1530 rpm on the V161 engine installed in Odeillo, operating with hydrogen at a mean pressure of 14 MPa and temperature ratio of 0.297, is compatible with a mechanical efficiency of 82% and an alternator efficiency of 92.5%.

Therefore, the analyses show that both prototypes have an advanced degree of development as evidenced by the similar, relatively high efficiencies of each subsystem.

Nevertheless, the engines are not dynamically similar. From this point of view, the differences between the V160 and V161 engines are not important in terms of geometric parameters, temperatures or pressures, but it is remarkable how much is influenced by the type of gas. Consequently, the indicated power losses caused by irreversibilities are greater in the V160 engine than in the V161, whose operating point corresponds to a characteristic Mach number a third smaller. In addition, the V161 engine coefficients of indicated power losses are so low that the characteristic Mach number corresponding to the maximum indicated power point reaches one of the highest values observed so far ($N_{MA,max} \approx 0.0060$). On the other hand, the use of hydrogen increases the characteristic Stirling number of the V161 engine, which allows moderate mechanical losses to be achieved at pressures even higher than those of the V160 engine.

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